

A-AS PHYSICS

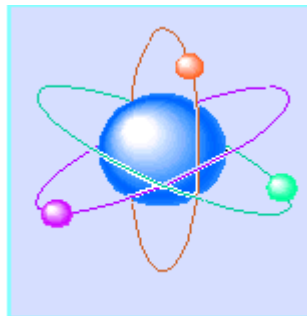
AS TOPIC-3B

Solid

Materials

B1-Investigating Materials

B2-Engineering Materials



First Edition

By

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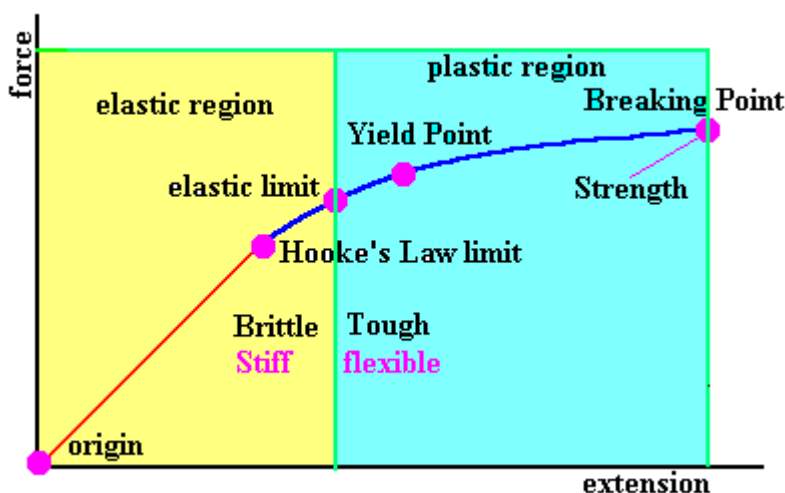
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Solid Materials

1.1 Investigating materials

1.1.1 Force-extension graphs

When a **stretching force** is applied to **solid material**, the material **length** in the force direction will increase. The difference between **original length** and **extended length** is called **extension**. The plot of various forces against their extension to a material is called “**Force-extension graph**”. In this graph, the y-axis represents the force and the x-axis represents the extension. The shape of the graph depends on the mechanical properties of the solid material. The general form of the graph is shown in figure below.



The graph is divided into five segments and five points.

Graph Points

1. **Hook's law limit** or limit of proportionality. This is the point where the material moves from **linear elastic deformation** to **non-linear elastic deformation**.
2. **Elastic limit**. This is the point where the material moves from **elastic deformation** to **plastic deformation**.
3. **Yield Point**. If the stress is increased beyond the elastic limit, a point is reached at which there is a marked increase in strain. Such a point is called the **yield point**.
4. **Breaking point**. At this point **ductile fracture** occurs.

Graph Segments

1. **Linear region, Hooke's region.**

This is a straight line that starts and passes through the origin to Hooke's law limit.

- a. The force is linearly proportional to extension. (i.e. if we double the force, the extension is doubled).
- b. The material in this region suffers **elastic deformation**.
- c. The material returns to its original length when the force is removed and the extension is zero. The recovery is complete.
- d. Hooke's law is obeyed in this region.

2. **Elastic non-linear region.**

This is a curved line that starts from Hooke's law limit to the elastic point.

- a. The force is not linearly proportional to extension. (i.e. if we double the force, the extension is not doubled).
- b. The material in this region suffers **elastic deformation**.

- c. The material returns to its original length when the force is removed and the extension is zero. The recovery is complete.
 - d. Hooke's law is not obeyed in this region.
3. **Inelastic Region or plastic region.**
 This is a curved line that starts from elastic limit point to the breaking point.
- a. The force is not linearly proportional to extension. (i.e. if we double the force, the extension is not doubled).
 - b. The material in this region suffers **inelastic deformation** or **plastic deformation** or a **permanent deformation**.
 - c. The material does not return to its original length when the force is removed and the recovery is no longer complete.
 - d. Hooke's law is not obeyed in this region
 - e. In this region the material is said to be **work-harden**.

1.1.2 Elastic and plastic behavior

The particles of a solid are held together electric forces of attraction. When external forces are applied to a solid, its shape changes and **deformation** occurs, which changes the lays structure of the solid building-units. There are two types of deformations: **reversible deformation (elastic deformation)** and **permanent deformation (plastic deformation)**.

1.1.2.1 Reversible deformation

If the deformation is **reversible (elastic) deformation**, then the material returns to its original shape when the forces on it are removed. For example: rubber returns to its original shape when stretched.

1.1.2.2 Permanent deformation

If the deformation is **permanent (plastic) deformation**, then the material does not return to its original shape when the forces on it are removed. For example: Plasticine takes on a new shape when stretched.

1.1.3 Stress and strain

The extension produced in a sample of a solid material depends on 1- the nature of the material, 2- the stretching force, 3- the cross-sectional area of the sample, and 4- its original length. To enable fair comparison between samples having different sizes, the terms '**stress**' and '**strain**' will be used when referring to the deforming force and the deformation it produces.

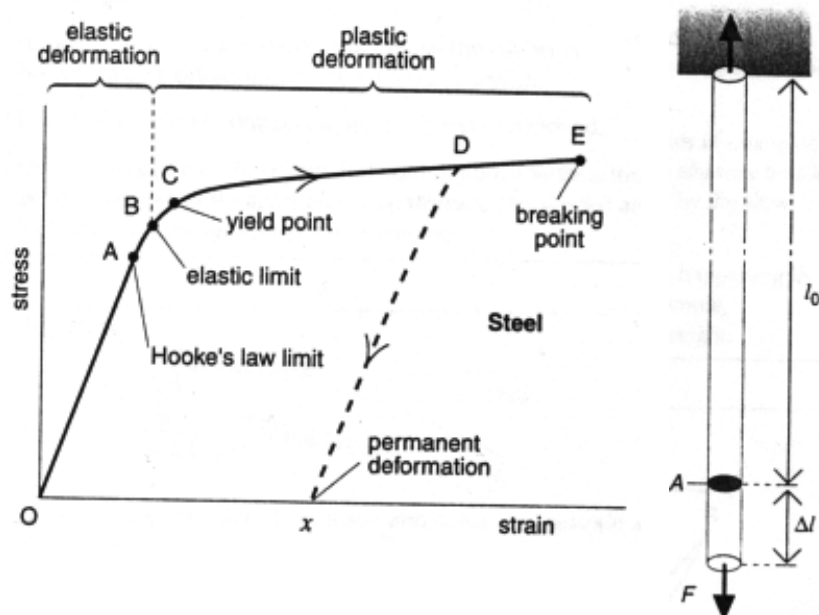
Stress, denoted by " σ " (sigma) is defined as force per unit area. That is, stress=force/area or $\sigma=F/A$, where "F" is force in Newton and 'A' area in m^2 and " σ " is stress in Pascal.

Strain, denoted by " ϵ " (epsilon) is defined as the extension a unit length (1m). That is, strain=extension/original length or $\epsilon=e/l=\Delta l/l$, where "e" is extension in meter ($e=\Delta l$ change in length) and 'l' is original length in meter. Note that strain is a ratio and has no unit.

If stress increases the length of the solid, then sample is in **tension** and the stress it is called "**tensile stress**" and the corresponding strain is also called "**tensile strain**". If stress decreased the length of the solid, then sample is in **compression** and the stress it is called "**compressive stress**" and the corresponding strain is also called "**compressive strain**".

1.1.4 Stress-strain Graphs

To compare between samples having different sizes, we plot 'stress' versus 'strain' and call the resulting curve "stress-strain graph". In this graph, the y-axis represents the stress and the x-axis represents strain. The shape of the graph depends on the mechanical properties of the solid material. The general form of the graph is shown in figure below.



The graph is divided into five segments and five points.

Graph Points

5. **Hook's law limit** or limit of proportionality. This is the point where the material moves from **linear elastic deformation** to **non-linear elastic deformation**.
6. **Elastic limit**. This is the point where the material moves from **elastic deformation** to **plastic deformation**.
7. **Yield Point**. If the stress is increased beyond the elastic limit, a point is reached at which there is a marked increase in strain. Such a point is called the **yield point**.
8. **Breaking point**. At this point **ductile fracture** occurs.

Graph Segments

4. **Linear region, Hooke's region.**

This is a straight line that starts and passes through the origin to Hooke's law limit.

- a. The force is linearly proportional to extension. (i.e. if we double the force, the extension is doubled).
- b. The material in this region suffers **elastic deformation**.
- c. The material returns to its original length when the force is removed and the extension is zero. The recovery is complete.
- d. Hooke's law is obeyed in this region.

5. **Elastic non-linear region.**

This is a curved line that starts from Hooke's law limit to the elastic point.

- a. The force is not linearly proportional to extension. (i.e. if we double the force, the extension is not doubled).
- b. The material in this region suffers **elastic deformation**.
- c. The material returns to its original length when the force is removed and the extension is zero. The recovery is complete.
- d. Hooke's law is not obeyed in this region.

6. **Inelastic Region or plastic region.**

This is a curved line that starts from elastic limit point to the breaking point.

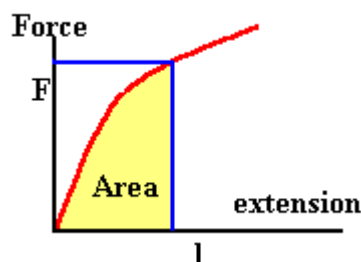
- The force is not linearly proportional to extension. (i.e. if we double the force, the extension is not doubled).
- The material in this region suffers **inelastic deformation** or **plastic deformation** or a **permanent deformation**.
- The material does not return to its original length when the force is removed and the recovery is no longer complete.
- Hooke's law is not obeyed in this region
- In this region the material is said to be **work-harden** or **strain-harden**.

1.1.5 Young's Modulus

The stress-strain curve for stretching a solid material over almost all elastic-region is a straight line that passes through the origin. That is, the tensile stress is linearly proportional to tensile strain during elastic deformation. This statement is called Hooke's law. In mathematical terms, stress/strain=constant. This constant is called **Young's Modulus** and is denoted by "E". So, $E = \text{stress}/\text{strain} = (F/A)/(e/l) = Fl/Ae$. Its value is the slope of the straight part of the stress-strain graph and it has the unit of Pascal.

1.1.6 Energetics of stretching

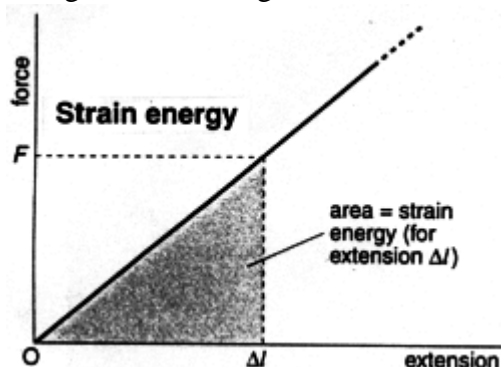
The graph shown below shows how extension varies with force for a material. Shaded area gives the work done for the extension " Δl ".



As work done on the material, the material stores energy. This energy is called "**strain energy**" and it is equal to the area under the curve.

1.1.7 Elastic Strain Energy

The graph shown below shows how extension varies with force for a material that obeys Hooke's Law. Shaded area gives the work done for the extension " Δl ". The area of a triangle = $\frac{1}{2}$ base height. So, the work done is $W = \frac{1}{2} F\Delta l$



As work done on the material, the material stores energy. This energy is called "**strain energy**". So, strain energy = $\frac{1}{2} F\Delta l$.

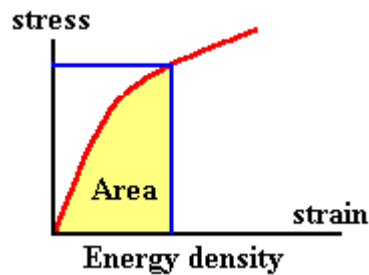
If " l " is the original length of the wire and " A " is its cross-sectional area, then the volume of wire is $V = Al$.

Strain energy = $\frac{1}{2} F \Delta l$. But from $\epsilon = \Delta l / l$. So, strain energy = $\frac{1}{2} F \epsilon l$. But from volume, $l = V / A$. So, strain energy = $\frac{1}{2} F \epsilon V / A = \frac{1}{2} F / A \epsilon V = \frac{1}{2} \sigma \epsilon V$. Divide the equation by “v” we get strain energy / $V = \frac{1}{2} \sigma \epsilon$. Thus, strain energy per unit volume = $\frac{1}{2} \sigma \epsilon$. That is,

$$\text{Strain energy per unit volume} = \frac{1}{2} \text{ stress} \times \text{strain}$$

1.1.8 Energy density

The term “strain energy per unit volume” is called “energy density” and it is equal to the area under the “stress-strain graph”.

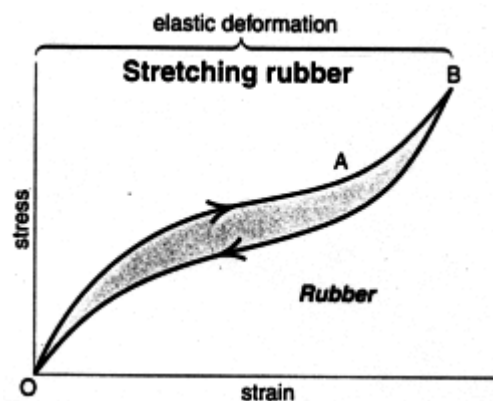


1.1.9 Stress-Strain graphs of special materials

1.1.9.1 Stretching Rubber

The graph shows what happens if increasing stress is applied to a rubber cord and then released before the breaking point. Rubber does not obey Hooke’s law. Also, much higher strains are possible than in steel and in glass. For example, if the extension is twice the original length then the strain is 2.

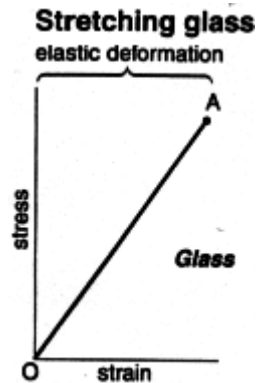
1. OA: The molecular chains in the rubber are being uncoiled and straightened.
2. AB: The chains are almost straight, so the rubber is becoming proportionately more difficult to stretch.
3. BA: The rubber contracts when the stress is removed



During this cycle of extension and contraction, energy is lost as heat. The effect is called “elastic hysteresis”. The shaded area represents the energy lost per unit volume.

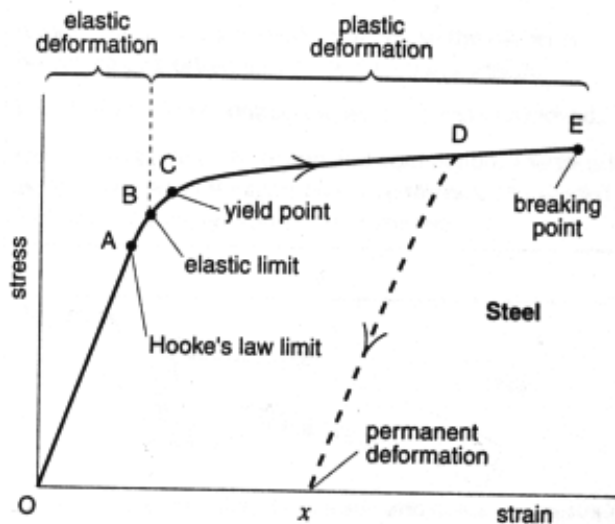
1.1.9.2 Stretching Glass and High carbon steel

The graph shown shows what happens if increasing tensile stress is applied to a glass thread or high carbon steel. Elastic deformation occurs until, at point A, a crack suddenly grows, and the glass breaks. A material that behaves like this is said to be **brittle**. The break is called a **brittle fracture**.



1.1.9.3 Stretching Metals: Copper, Mild Steel

Unlike glass, most metals do not experience brittle fracture when stretched because dislocations tend to stop cracks growing and spreading. The stress-strain graph for steel is shown below.



1. OB represents the deformation of the wire is elastic
2. B is the elastic limit. Beyond it, the deformation becomes plastic as layers of particles slide over each other. If the stress were removed at, say, point D, the wire would be left with a permanent deformation.
3. C this is the yield point. Beyond it, little extra force is needed to produce a large extra extension. If a material can be stretched beyond the yield point, then it is said to be **ductile**.
4. E the wire develops a thin "neck", and then a **ductile fracture** occurs. The highest stress just before the wire breaks is called the "**ultimate tensile stress**".

1.1.10 Describing the behavior of materials

The behavior of solid materials under stress determines their mechanical properties. The following terminologies are used to describe mechanical properties.

1. **Strength** deals with how great an applied force a material can withstand without breaking. A strong material has a high ultimate tensile stress. (i.e. glass, steel)
2. **Stiffness** describes the ability of a material to resist deformation. A stiff material is not very flexible. A stiff material has a high Young's modulus. A high stress produces little strain. (i.e. glass, steel)

- Ductility** or **workability** describes the ability of a material to be hammered, pressed, bent, rolled, cut or stretched into useful shapes. A ductile material usually can be drawn into wires (i.e. steel, iron, copper)
- Toughness** describes the material ability to deform plastically before it breaks. It describes the ability to resist cracks. A tough material is not brittle and cracks are not easily spread.
- Malleability** describes the ability of a material to be hammered, pressed, bent, rolled, cut or stretched into useful shapes. (i.e. steel, iron, copper)

6. Examples

| Material | Strong | Stiff | Ductile | Tough | Malleability |
|----------|--------|-------|---------|-------|--------------|
| Glass | ✓ | ✓ | ✗ | ✗ | ✗ |
| Iron | ✓ | ✓ | ✓ | ✓ | ✓ |
| Steel | ✓ | ✓ | ✓ | ✓ | ✓ |
| Copper | ✓ | ✓ | ✓ | ✓ | ✓ |
| Rubber | ✓ | ✗ | ✓ | ✓ | ✓ |
| Diamond | ✓ | ✓ | ✗ | ✓ | ✗ |
| Nylon | ✗ | ✗ | ✓ | ✓ | ✓ |
| Putty | ✗ | ✗ | ✗ | ✗ | ✓ |

1.1.11 Questions

- Why stress and strain are used rather than force and extension to describe the deformation of solids?
- The young modulus for steel is greater than that of brass. Which would stretch more easily? Which is stiffer?
- How does a deformed body behave when the deformation force is removed if the strain is 1- elastic, 2- plastic?
- Define elasticity, stress, strain, elastic limit, yield point, Hooke's law limit, elastic hysteresis, brittle, strain-harden,
- Define strength, stiffness, toughness, ductility, and Malleability

1.1.12 Problems

- An aluminum wire of length 0.35 mm and a radius of 0.20 mm is stretched by 1.44 mm. Knowing that the Young's modulus is 7×10^{10} Pa find the ...
 - Strain in wire
 - Stress in wire
 - Cross-sectional area of the wire
 - Tension in the wire
- A length of copper square cross-section measuring 1 mm by 1 mm is stretched by a tension of 40N. What is the tensile stressing Pa?

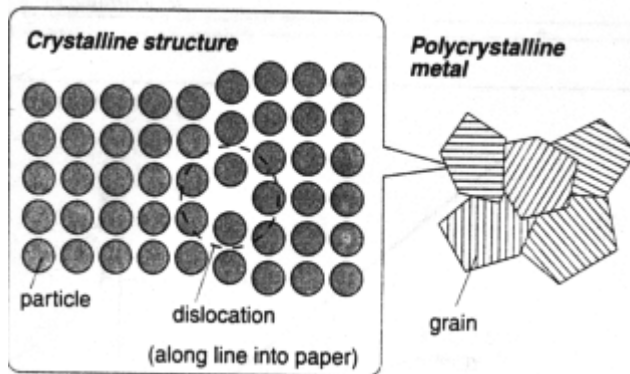
3. If the breaking stress of steel is 1×10^9 Pa, will a wire of the same material of cross-section area 4×10^{-4} cm² break when a 10kg mass is hung from it?
4. A strip of rubber 6 cm long is stretched until it is 9cm long. What is the tensile strain in the rubber as 1- a ratio, 2- percentage?
5. A wire originally 2m long suffers a 0.1% strain. What is its stretched length?
6. A brass wire 2.5 m long of cross-section area 1×10^{-3} cm² is stretched 1 mm by a load of 0.4 kg. Calculate the Young's modulus "E" for brass. What percentage strain does the wire suffer? Use the value of "E" to calculate the force required to produce a 4% strain in the same wire. Is your answer for the force is reliable? If it were not, would it be greater or less than your answer?
7. In problems of chapter-11 which starts in page-213 solve B34, B43, B46, B48, B53, B56, B60,

1.2 Engineering Materials

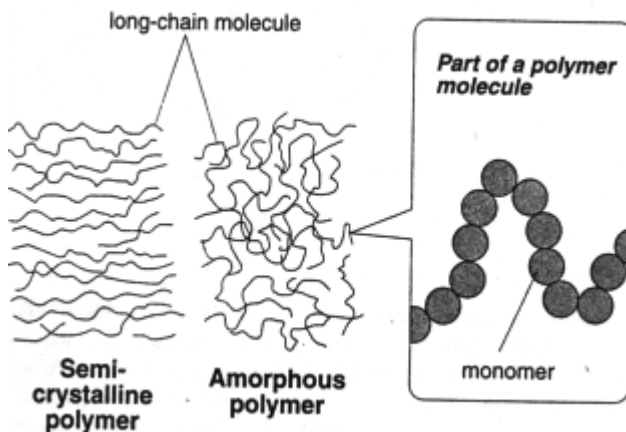
1.2.1 Polycrystalline materials

Based on their particles arrangements, solids can be classified into three main types: **Crystalline solids**, **Amorphous (glassy) Solids**, and **Polymers**.

1. **Crystalline solids.** The particles are in regular, repeating pattern. They may form a single crystal, as with diamond. However, they may be millions of tiny crystals joined together to form **grains**. Most metals have this many grains and thus have a **polycrystalline structure**.



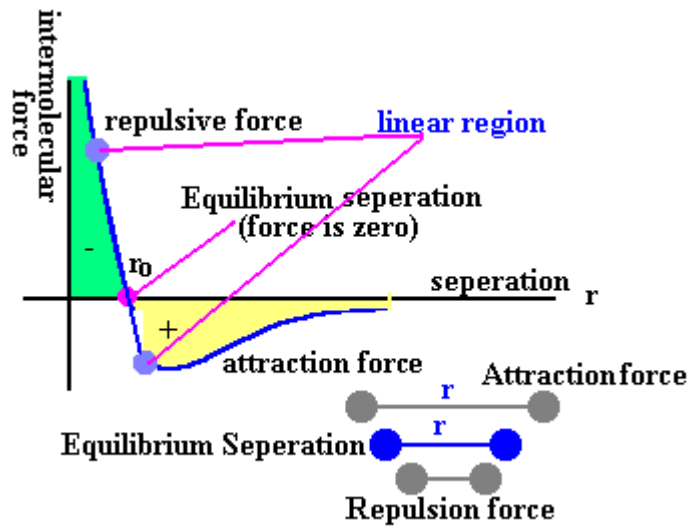
2. **Amorphous (glassy) Solids.** The particles have no regular pattern (except over a very short distances). Glass and wax have structure like this.
3. **Polymers.** These materials have long-chain molecules, each of which may contain many thousands of atoms. The molecules are formed from the linking of short units called “**monomers**”. In a polymer, the chain may be coiled up and tangled like spaghetti. Depending on the amount of tangling, a polymer may be described as **semi-crystalline** or **amorphous**.



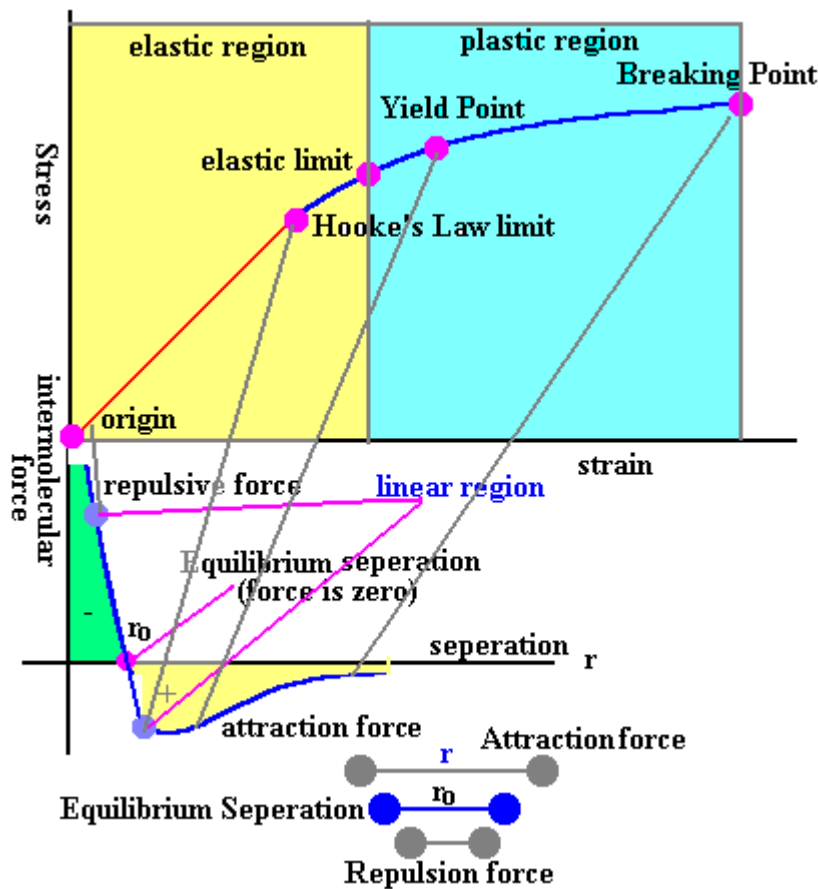
Note that rubber and wool are natural polymers. Plastic, such as nylon and artificial rubber, are synthetic polymers.

1.2.1.1 Molecular structure

The physical properties of materials depend on how their atoms or molecules are arranged. Assume we have two atoms separated by a distance “ r ”. The figure to the right shows how the intermolecular force between the two atoms varies with separation “ r ”. Positive intermolecular force means a repulsion and negative intermolecular force means attraction.

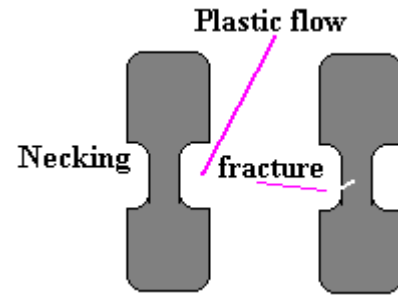


Based on the intermolecular force we can justify the shape of stress-strain graph. For example, for metals the elastic region is almost linear and it is exactly linear for Hooke’s region. Imagine that the atoms of a metal are spaced at r_0 and to be further separated by stress. Since the graph of intermolecular force is linear (straight) in this region, the increase in separation is linear and proportional to the force. This is reflected at the macroscopic level in Hooke’s law.



1.2.1.2 Plastic flow, necking and fracture

An important feature of tensile tests on most metals at very low temperatures is the occurrence of unstable "discontinuous" plastic deformation, i.e., sudden load drops associated with local elongations, which follow sudden local increases in temperature. Associated with each load drop is the formation of a localized "neck", a region of slightly smaller cross-sectional area, in the test specimen. At this point, an additional load will cause a fracture.

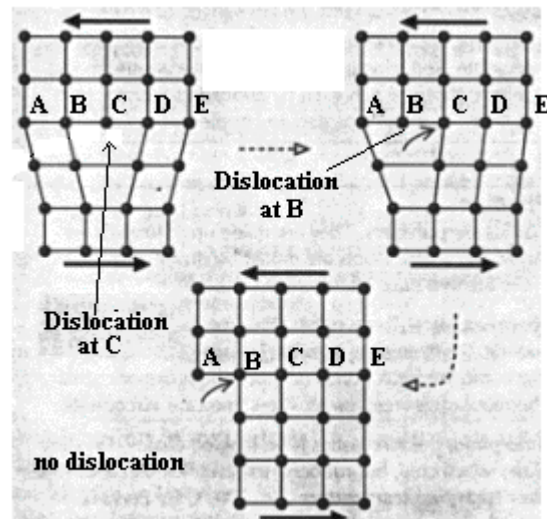


1.2.2 Defects in crystals

Defects occur in all crystals (e.g. grains). There are two types of defects: **line defect** and **point defect**. Line defects are the dislocations. They are associated with stacking faults.

Edge dislocations occur due to growth faults during crystallization of metals where there may be an incomplete plane of atoms.

When a wire is stretched plastically, layers of atoms slip over each other. With dislocations present, much less force is needed to cause slipping. The figure to the right shows a dislocation at point "C". When the crystal is moved to the right, the bond at "B" is broken and it is formed at point "C". The broken bond at "B" leaves a dislocation at point "B". When the crystal is moved to the right one more time, the bond at "A" is broken and it is formed at point "B". Thus, the final crystal structure does not have dislocations.



Grain boundary exists in most metals because they are polycrystalline. They consist of large group of small crystals called **grains**. Because grains have different structure of crystals, the boundary between any two grains is imperfect and can act as an obstacle to dislocation-movements. In general, the smaller the grains the more difficult it is to deform the metal.

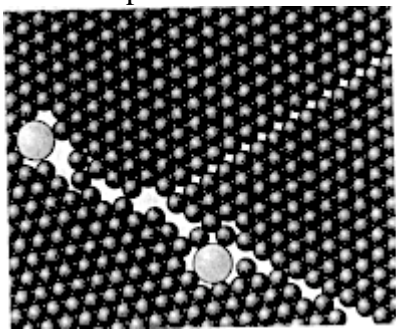
Work hardening occurs when a metal is hammered, stretched, or bent. It produces more dislocations but these become so jumbled that they block the spread of dislocations.

Strength of metals

A metal in which there were no dislocations would be extremely strong. So, metals can be made stronger by controlling the movement of dislocations as follows:

1. Increase the number of dislocations. This will make it more likely that the various dislocations will obstruct each other in a 'log jam' effect. The number of dislocations can be increased by a process called "**work-hardening**".

2. Introducing 'foreign' atoms into the structure (e.g. carbon atoms in iron to form steel). This will prevent the movement of dislocations.



3. Dislocations have difficulties in moving across grain boundary and therefore samples in which the grain size is small tend to be stronger.

1.2.3 Heat treatment: annealing, quenching, tempering

Heat treatment is aimed on improving the properties of metals. There are three methods of heat treatment: **annealing**, **quenching**, and **tempering**.

Annealing is a process by which a metal is heated and cooled very slowly to make it more flexible. Annealing reduces the number of dislocations in the metal. (i.e. used in copper and steel)

Quenching is a process by which a metal is heated to high temperature and then rapidly cooled to retain mechanical properties associated with a crystalline structure that would be lost if cooled slowly. Commonly applied to steel.

Tempering is a process by which a metal is heated to high temperature below the melting point and then cooled by air. This process has the effect of toughening a material by reducing its brittleness and internal stresses.

1.2.4 Failure mechanism: cracks, fatigue, creep

Cracks. External and internal cracks play an important part in the fracture of a material and prevent it from displaying its theoretical strength.

Fatigue. If a metal is taken through many cycles of changing stress, a fatigue fracture may occur before the ultimate tensile stress is reached. Fatigue fractures are caused by the slow spread of small cracks.

Creep. This is the deformation, which goes on happening in some materials if stress is maintained. For example, unsupported lead slowly sags under its own weight.

1.2.5 Thermoset and thermoplastic polymers

Plastic are polymers. They are made up of long-chain molecules whose atoms are linked by strong covalent bonds. There are two main classes of plastic: **thermoplastic** and **thermosets**. The thermoplastic is divided into two classes: **amorphous thermoplastic**, and **semi-crystalline thermoplastic**. Thermosets are divided into two classes: **rigid thermosets** and **Elastomer thermosets**.

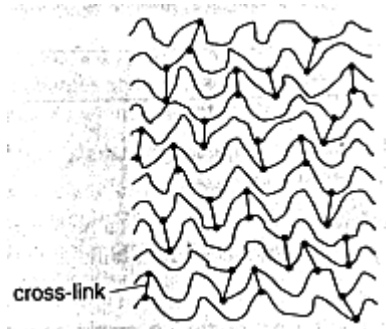
1- Thermoplastic

- a) Thermoplastic are soften when heated and harden on cooling

- b) Re-softening is possible because thermal activity can overcome the weak bonds between polymers chains.
- c) Can creep under stress and are called **viscoelastic**.
- d) **Amorphous thermoplastic**
 - i) They are thermoplastic with tangled chains
 - ii) They are glassy when cold but rubbery (soft and flexible) above their **glass transition temperature**. (e.g. 100C for Perspex)
 - iii) E.g. Perspex
- e) **Semi-crystalline thermoplastic**
 - i) Have regions where the chains are parallel and close.
 - ii) Chain bonds are strong.
 - iii) Their strong bonds, produce stiffness and good tensile strength
 - iv) E.g. polythene nylon

2- **Thermosets**

- a) They do not soften when warmed, thus they cannot be remolded
- b) During manufacturing they develop strong and permanent cross-links between their chains.



c) **Rigid Thermosets**

- i) E.g. artificial rubber

d) **Elastomer Thermosets**

- i) e.g. epoxy resins and Melamine

| Summary Table On Plastics | | | | |
|---------------------------|----------------------|----------------------|-------------------|--------------------------|
| | Thermoplastic | | Thermosets | |
| Property | resoften when wormed | | Permanent set | |
| Type | Amorphous | Semi-crystalline | Elastomer | Rigid |
| Structure | Tangled chains | Many parallel Chains | Some cross links | Many cross links |
| Examples | Perspex | Polythene Nylon | Artificial rubber | Epoxy resins Melamine |

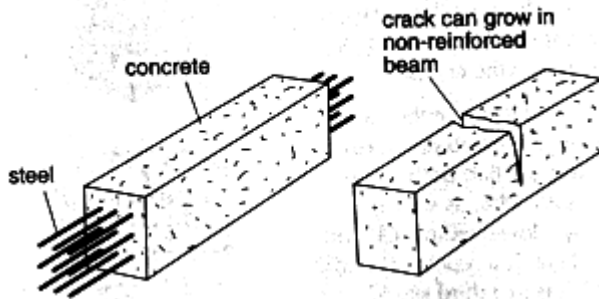
1.2.6 Elastomers

Elastomer is a big fancy word, and all it means is "rubber". Some polymers that are elastomers include artificial and natural rubber. What makes elastomers special is that they can be stretched to many times their original length, and can bounce back into their original shape without permanent deformation. Cross-linked polymers can't be recycled very easily but thermoplastic elastomers can be recycled.

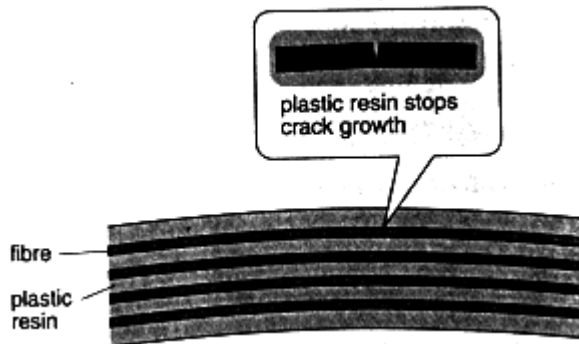
1.2.7 Composite materials

Materials can be composed together to make use of best properties of each.

1. Reinforced concrete. Concrete has high compressive strength but is brittle and weak in tension. To prevent its weakness in tension, it can be reinforced with steel rods. In a pre-stressed concrete, the rods are stretched elastically before the concrete sets. This gives greater strength and stiffness.



2. Glass reinforced plastic called "**fiber glass**". Glass fibers are embedded in plastic resin. The fiber provides tensile strength and the resin provides stiffness. Thus, glass reinforced plastic is strong and stiff.



3. Carbon fiber reinforced plastic called "carbon-fiber" is similar to fiberglass but with stronger, stiffer carbon fibers instead of glass.
4. Laminates are discrete layers reinforced by other materials such as fibrous mats, woven cloth, ...etc.
5. Plywood is a form of sandwich construction of natural wood fibers with plastic.
6. Chipboard is a paperboard used for backing and making cartons. It is made of mixed, unbleached paper stocks coated with Manila paper.
7. Fiber reinforced polymers
8. Fiber and particle composites

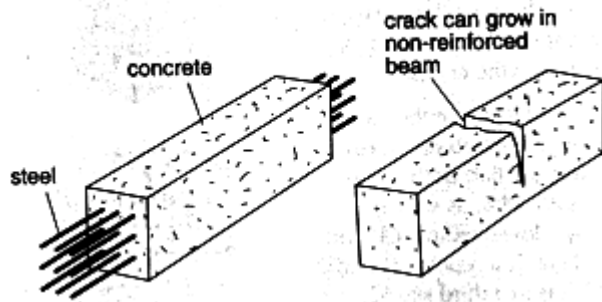
1.2.8 Bridging a space with beams

1.2.8.1 Steel tension members

Steel members are used to construct bridges. They come in the form of “L” shape, “I” shape, or rectangular shape.

1.2.8.2 Pres-stressed reinforced concrete

Concrete has high compressive strength but is brittle and weak in tension. To prevent its weakness in tension, it can be reinforced with steel rods. In a pre-stressed concrete, the rods are stretched elastically before the concrete sets. This gives greater strength and stiffness.



1.2.8.3 Applications of principle of moments to systems of non-parallel forces

If an object is in equilibrium, then the resultant of all forces acting on an object must be zero and the sum of all moments at any point in the system is zero.

$$1- \Sigma F_x = 0, -T_1 \sin 30 + T_2 \sin 60 = 0, -0.5T_1 + 0.866T_2 = 0$$

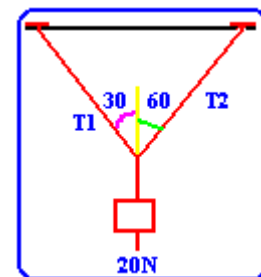
$$T_1 = 1.73T_2$$

$$2- \Sigma F_y = 0, T_1 \cos 30 + T_2 \cos 60 - 20 = 0, 0.866T_1 + 0.5T_2 = 20$$

$$\text{From (1) } 0.866(1.73T_2) + 0.5T_2 = 20, 1.50T_2 + 0.5T_2 = 20$$

$$T_2 = 20/2 = 10 \text{ N}$$

$$T_1 = 1.73T_2 = 17.3 \text{ N}$$



A uniform rod AB of length $4a$ and weight W is smoothly hinged at its upper end, A. The rod is held at 30° to the horizontal by a string which is at 90° to the rod and attached to it at C where $AC = 3a$. Find (a) the tension in the string, (b) the vertical component of the reaction force, (c) the horizontal component of the reaction at A. (problem-9/49)

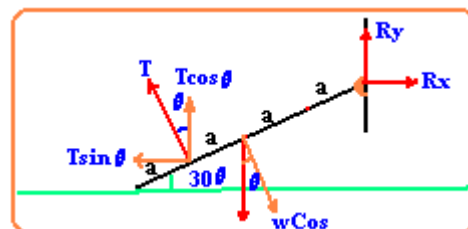
$$1- (\Sigma \tau)_A = 0: T \cdot 2a - 2a \cdot W \cos 30 = 0$$

$$T = (2/3)W \cos 30 = 0.58W$$

$$2- \Sigma F_y = 0: R_y + T \cos 30 - w = 0$$

$$R_y = W - T \cos 30 = W(1 - 0.58 \cos 30) = 0.5W$$

$$R_x = T \sin 30 = 0.5W \sin 30 = 0.29W$$



1.2.8.4 Applications of principle of moments to systems of parallel forces

1. **What is meant by Coplanar forces?**
Coplanar forces are forces whose line of action never intersects.
2. **What is the physical meaning of torque (moment)?**
Torque is a measure of the effectiveness of a force in producing rotation about an axis.
3. **What is the definition of torque?**
Torque is defined to be the product of the force and the perpendicular distance from the axis of rotation to the line of action of the force. Mathematically, torque=(force)(lever-arm), symbolically, $\tau = f \cdot d$
4. **What is meant by level-arm or moment arm?**
It is the perpendicular distance from the axis of rotation to the line of action of the force.
5. **What are the equilibrium conditions under coplanar forces?**
1) $\Sigma F = 0$: 1- $\Sigma F_x = 0$ 2- $\Sigma F_y = 0$ 3- $\Sigma F_z = 0$
2) $(\Sigma \tau)_A = 0$
6. **States the principle of moments?**
The sum of moments at any point in the system is zero. $(\Sigma \tau)_A = 0$
7. **What are the directions of torque?**
A torque that tends to cause a clockwise rotation is **negative**
A torque that tends to cause a counterclockwise rotation is **positive**
8. **What is meant by Center of Gravity?**
Is the point at which the entire weight of the object may be considered concentrated.
9. **How to solve coplanar equilibrium problems:**
1- Draw a free body diagram
2- Replace weights by an equivalent force at the center mass
3- Show reaction forces at joints
4- Apply force conditions $\Sigma F = 0$: 1- $\Sigma F_x = 0$ 2- $\Sigma F_y = 0$ 3- $\Sigma F_z = 0$
5- Apply torque condition $(\Sigma \tau)_A = 0$ at a point with the maximum number of unknown forces.

Problems

1. **A uniform pole AB of weight 5W and length 8a is suspended horizontally by two vertical strings attached to it at C and D where AC=DB=a. A body of weight 9W hangs from the pole at E where ED=2a. Calculate the tension in each string. (problem 6/49)**

$$1 - \Sigma F_y = 0: T_c + T_d - 9W - 5W = 0$$

$$2 - (\Sigma \tau)_d = 0: T_c 6a - 9W 2a - 5W 3a = 0$$

$$T_c = 33W/6 = 5.5W$$

$$T_d = 9W + 5W - T_c = 14W - 5.5W = 8.5W$$



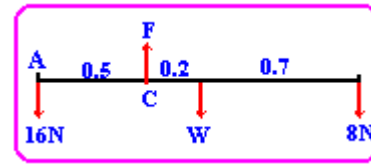
2. AB is a uniform rod of length 1.4m. It is pivoted at C, where AC=0.5m, and resets in horizontal equilibrium when weights of 16N and 8N are applied at A and B respectively. Calculate: (a) the weight of the rod, (b) the magnitude of the reaction at the pivot. (problem-7/49).

$$1- \Sigma F_y=0: -16+F-W-8=0$$

$$2- (\Sigma \tau)_c=0: 16*0.5-w*0.2-8*0.9=0$$

$$W=(8-0.2W-7.2) /0.2=0.8/0.2=4N$$

$$F=16+4+8=28N$$



Note: Study chapter nine (page 142) and eleven.(page 181)